NONMEASURABLE UNIONS WITH RESPECT TO TREE IDEALS

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Let $T$ be a tree $T \subseteq \omega^{<\omega}$. Body of $T$ is $[T] = \{x \in \omega^{\omega} : \forall n \ x|n \in T\}$.

A tree $T$ is called

• perfect or Sacks if $(\forall \sigma \in T)(\exists \tau \in T)(\sigma \subseteq \tau \land (\exists n \neq m)(\tau \downarrow n, \tau \downarrow m \in T));$
• superperfect or Miller if $(\forall \sigma \in T)(\exists \tau \in T)(\sigma \subseteq \tau \land (\exists^{\omega} n)(\tau \downarrow n \in T));$
• Laver if there is a node $s \in T$ such that, for every node $t \in T$ if $s \subseteq t$ then $t$ is infinitely splitting i.e. $\{n \in \omega : s \downarrow n \in T\}$ is infinite.

Let $\mathcal{T}$ be a family of trees. We say that $A \in P(\omega^{\omega})$ is in $t_0$ iff

$(\forall P \in T)(\exists Q \in T) \ Q \subseteq P \land [Q] \cap A = \emptyset.$

We say that $A \in P(\omega^{\omega})$ is $t$-measurable iff

$(\forall P \in T)(\exists Q \in T) \ Q \subseteq P \land ([Q] \subseteq A \lor [Q] \cap A = \emptyset).$

The first result is connected to $s_0$ ideal i.e. a classic Marczewski ideal. Let $A \subseteq s_0$ be point-finite family of subsets of $\omega^{\omega}$ such that $\bigcup A \notin s_0$. Then there is a subfamily $A' \subseteq A$ such that $\bigcup A'$ is not $s$-measurable.

Analogous result is true in the case of Miller ideal $m_0$ and $m$-measurability.

We also show that it is relatively consistent with ZFC that there is $\omega_1$-point family $A \subseteq s_0 \cap l_0 \cap m_0$ such that $\bigcup A = \omega^{\omega}$ and union of any subfamily of $A$ is $I$-measurable where $I \in \{s_0, l_0, m_0\}$.

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