NONMEASURABLE UNIONS WITH RESPECT TO TREE IDEALS

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Let T be a tree $T \subseteq \omega^{<\omega}$. Body of T is $[T] = \{x \in \omega^\omega : \ \forall n \ x|n \in T\}$.

A tree T is called

- *perfect* or *Sacks* if $(\forall \sigma \in T)(\exists \tau \in T)(\sigma \subseteq \tau \wedge (\exists n \neq m)(\tau^\frown n, \tau^\frown m \in T)$;
- *superperfect* or *Miller* if $(\forall \sigma \in T)(\exists \tau \in T)(\sigma \subseteq \tau \wedge (\exists^\infty n)(\tau^\frown n \in T)$;
- *Laver* if there is a node $s \in T$ such that, for every node $t \in T$ if $s \subseteq t$ then t is infinitely spliting i.e. $\{n \in \omega : \ s^\frown n \in T\}$ is infinite.

Let $\mathcal{T}$ be a family of trees. We say that $A \in P(\omega^\omega)$ is in t_0 iff

$$(\forall P \in \mathcal{T})(\exists Q \in \mathcal{T}) \ Q \subseteq P \wedge [Q] \cap A = \emptyset.$$

We say that $A \in P(\omega^\omega)$ is t-measurable iff

$$(\forall P \in \mathcal{T})(\exists Q \in \mathcal{T}) \ Q \subseteq P \wedge ([Q] \subseteq A \vee [Q] \cap A = \emptyset).$$

The first result is connected to s_0 ideal i.e. a classic Marczewski ideal. Let $\mathcal{A} \subseteq s_0$ be point-finite family of subsets of ω^ω such that $\bigcup \mathcal{A} \notin s_0$. Then there is a subfamily $\mathcal{A}' \subseteq \mathcal{A}$ such that $\bigcup \mathcal{A}'$ is not s-measurable.

Analogous result is true in the case of Miller ideal m_0 and m-measurability.

We also show that it is relatively consistent with ZFC that there is ω_1-point family $\mathcal{A} \subseteq s_0 \cap l_0 \cap m_0$ such that $\bigcup \mathcal{A} = \omega^\omega$ and union of any subfamily of $\mathcal{A}$ is I-measurable where $I \in \{s_0, l_0, m_0\}$.

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